JOURNAL OF PROPULSION AND POWER
Vol. 13, No. 1, January–February 1997

Technical Comments

Comment on "Theoretical Upper Limits on Enthalpy Rocket Performance"

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THE article by Parker and Humble[1] examines the performance of enthalpy rocket propulsion based on the use of a thermal capacitor, which transfers its stored thermal energy to a nonreactive expellent. The article concentrates on the utilization of the energy released during a phase change (solidification of a molten material).

However, one may get the wrong impression that the calculated performance should be related to a specific engine, while, in fact, it is the intention of this Comment to show that the expected velocity increment ΔV, which was chosen to characterize the engine performance, is actually independent of engine size (e.g., overall mass m and throat area A_t), thrust F, or operating conditions (e.g., chamber pressure P_0, mass flow rate $\dot{m}$, energy transfer rate $\dot{H}$, or burn time t_b), and is merely a function of thermodynamic properties.

With the absence of gravity and atmospheric influence, the velocity increase during engine operation is related to the equivalent velocity and mass ratio

$$\Delta V = u_{\text{eq}}\, \ell n(m_0/m_b) \tag{1}$$

where m_0 is the initial mass and m_b is the final mass (at burnout). For an engine consisting of capacitor and expellent masses (m_c and m_e, respectively) only, one obtains:

$$\Delta V = u_{\text{eq}}\, \ell n[1 + (m_e/m_c)] \tag{2}$$

The overall capacitor energy (phase change energy) is equal to the total thermal energy transferred to the expellent gas

$$H_c = m_c \Delta Q_f = m_e c_p T_0 \tag{3}$$

where ΔQ_f is the capacitor material's latent heat of melting per unit mass, c_p is the expellent gas specific heat, and T_0 is its total temperature, which is equal to the melting temperature of the capacitor material.

For complete expansion of the expellent to vacuum, the exit velocity u_e is equal to the equivalent velocity u_{eq} and can be expressed by

$$u_{\text{eq}} = u_e = \sqrt{2c_p T_0} \tag{4}$$

Combining Eqs. (2–4) one obtains

$$\Delta V = \sqrt{2c_p T_0}\, \ell n[1 + (\Delta Q_f / c_p T_0)] \tag{5}$$

Equation (5) reveals the thermodynamic nature of the problem and the independence of ΔV on engine characteristics.

The following equation, which has been the concluding expression Eq. (8) of Ref. 1

$$\Delta V = F\left(\ell n\left\{\frac{\dot{H}}{\Delta Q_f} + \left[\frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{M\gamma}{R_u}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(1-\gamma)}}\right]\right\} - \ell n\left(\frac{\dot{H}}{\Delta Q_f}\right)\right)\Bigg/\left[\frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{M\gamma}{R_u}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(1-\gamma)}}\right] \tag{6}$$

obscures the important conclusion of Eq. (5).

In fact, one can show the identity of Eq. (6) [Eq. (8) in Ref. 1] with the expression presented here [Eq. (5)]. Substituting

$$F = \dot{m} u_{\text{eq}} \tag{7}$$

and bearing in mind that the mass flow rate through a choked nozzle is

$$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{M\gamma}{R_u}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(1-\gamma)}} \tag{8}$$

would convert Eq. (6) to

$$\Delta V = u_{\text{eq}}\{\ell n[(\dot{H}/\Delta Q_f) + \dot{m}] - \ell n(\dot{H}/\Delta Q_f)\} \tag{9}$$

Now, expressing $\dot{H}$ as

$$\dot{H} = \dot{m} c_p T_0 \tag{10}$$

Table 1 ΔV values for selected capacitor/expellent combinations as calculated from Eq. (5)

Capacitor material	Maximum ΔV, km/s		
	H_2	O_2	Ar
Li	0.343	0.864	0.917
B	0.396	1.331	1.538
Al	0.145	0.483	0.584
BeO	0.579	1.619	1.848
MgO	0.389	1.187	1.426
BeSi	0.546	1.393	1.511

Received May 17, 1996; revision received Aug. 1, 1996; accepted for publication Aug. 5, 1996.

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and using Eq. (4), it is obvious that Eq. (9) is actually identical to Eq. (5), which means that Eq. (8) of Ref. 1 is equivalent to Eq. (5) of this work, as was claimed before.

Applying Eq. (5) to the properties summarized in Table 1 of Ref. 1 for a number of capacitor materials and a selection of expellent gases yields the same trend presented in Table 2 of Ref. 1 with somewhat different quantitative results. Examples are given in Table 1.

In summary, this author would like to stress the thermodynamic nature of the enthalpy rocket performance as opposed to an engine operating characteristics dependence.

Reference

[1]Parker, T. W., and Humble, R. W., "Theoretical Upper Limits on Enthalpy Rocket Performance," *Journal of Propulsion and Power*, Vol. 12, No. 2, 1996, pp. 445–448.